SLAC–PUB–7272
August 1996

NONPERTURBATIVE RENORMALIZATION OF QED IN LIGHT-CONE QUANTIZATION*

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ABSTRACT

As a precursor to work on QCD, we study the dressed electron in QED nonperturbatively. The calculational scheme uses an invariant mass cutoff, discretized light-cone quantization, a Tamm–Dancoff truncation of the Fock space, and a small photon mass. Nonperturbative renormalization of the coupling and electron mass is developed.

Paper submitted to the
1996 Annual Divisional Meeting (DPF96) of the
Division of Particle and Fields of the
American Physical Society
University of Minnesota, Minneapolis, Minnesota
10–15 August 1996

* Work supported in part by the Minnesota Supercomputer Institute through grants of computing time and by the Department of Energy, contract DE–AC03–76SF00515.

1 Introduction

We are in the process of studying dressed fermion states in a gauge theory. To give the work specific focus, we concentrate on the nonperturbative calculation of the anomalous moment of the electron. [1] This is not intended to be competitive with perturbative calculations. [2] Instead it is an exploration of nonperturbative methods that might be applied to QCD and that might provide a response to the challenge by Feynman [3] to find a better understanding of the anomalous moment.

The methods used are based on light-cone quantization [4] and on a number of approximations. Light-cone coordinates provide for a well-defined Fock state expansion. We then approximate the expansion with a Tamm–Dancoff [5] truncation to no more than two photons and one electron. The Fock-state expansion can be written schematically as $\Psi = \psi_0|e\rangle + \psi_1|e\gamma\rangle + \psi_2|e\gamma\gamma\rangle$. The eigenvalue problem for the wave functions ψ_i and the bound-state mass M becomes a coupled set of three integral equations. To construct these equations we use the Hamiltonian $H_{\rm LC}$ of Tang *et al.* [6] The anomalous moment is then calculated from the spin-flip matrix element of the plus component of the current. [7] The regulator is an invariant-mass cutoff $\sum_i (P^+/p_i^+)\,(m_i^2 + p_{\perp i}^2) \leq \Lambda^2$. Additional approximations and assumptions are a nonzero photon mass of $m_e/10$, a large coupling of $\alpha = 1/10$, and use of numerical methods based on discretized light-cone quantization (DLCQ). [4]

2 Renormalization

We renormalize the electron mass and couplings differently in each Fock sector, as a consequence of the Tamm–Dancoff truncation. [8] The bare electron mass in the one-photon sector is computed from the one-loop correction allowed by the two-photon states. We then require that the bare mass in the no-photon sector be such that $M^2 = m_e^2$ is an eigenvalue.

The three-point bare coupling e_0 is related to the physical coupling e_R by $e_0(\underline{k}_i, \underline{k}_f) = Z_1(\underline{k}_f) e_R / \sqrt{Z_{2i}(\underline{k}_i) Z_{2f}(\underline{k}_f)}$, where $\underline{k}_i = (k_i^+, \mathbf{k}_{\perp i})$ is the initial electron momentum and $\underline{k}_f$ the final momentum. The renormalization functions $Z_1(\underline{k})$ and $Z_2(\underline{k}) = |\psi_0|^2$ are generalizations of the usual constants. The amplitude ψ_0 must be computed in a basis where only allowed particles appear.

The function Z_1 can be fixed by considering the proper part of the transition amplitude T_{fi} for photon absorption by an electron at zero photon momentum ($\underline{q} = \underline{k}_f - \underline{k}_i \to 0$): $T_{fi}^{\rm proper} = V_{fi}/Z_1(\underline{k}_f)$, where V_{fi} is the elementary three-point vertex. The transition amplitude can be computed from $T_{fi} = \psi_0 \langle \Psi | V | i \rangle$, in which $|\Psi\rangle$ is the dressed electron state and $\psi_0 = \sqrt{Z_{2f}(\underline{k}_f)}$. The proper amplitude is then obtained from $T_{fi}^{\rm proper} = T_{fi}/(Z_{2i} Z_{2f})$, where the Z_2's remove the disconnected dressing of the electron lines.

Thus the solution of the eigenvalue problem for only one state can be used to compute Z_1. Full diagonalization of $H_{\rm LC}$ is not needed. Because Z_1 is needed in the

construction of H_{LC}, the eigenvalue problem and the renormalization conditions must be solved simultaneously.

Most four-point graphs that arise in the bound-state problem are log divergent. To any order the divergences cancel if all graphs are included, but the Tamm–Dancoff truncation spoils this. For a nonperturbative calculation we need a counterterm $\sim \lambda(p_i^+, p_f^+) \log \Lambda$ that includes infinite chains of interconnected loops. The function λ might be fit to Compton amplitudes. [9] Thus we need to be able to handle scattering processes.

3 Preliminary Results and Future Work

Some preliminary results are given in Fig. 1. In the two-photon case there remain divergences associated with four-point graphs.

The next step to be taken in this calculation is renormalization of the four-point couplings, followed by numerical verification that all logs have been removed. Construction of finite counterterms that restore symmetries will then be considered. We can also consider photon zero modes, Z graphs, and pair states.

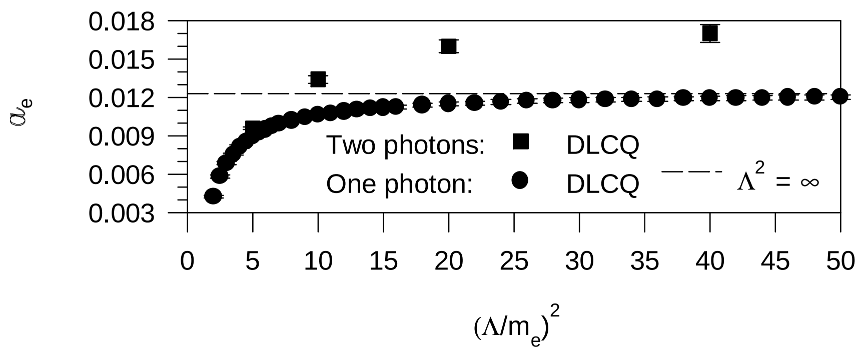

Figure 1: Electron anomalous moment as a function of the cutoff Λ^2, extrapolated from DLCQ calculations. The photon mass is $m_e/10$, and the coupling is 1/10.

Acknowledgments

This work has benefited from discussions with R. J. Perry, St. D. Głazek, and G. McCartor.

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